Prove that $\pi^2 \le \int_0^{\frac{\pi}{3}} 36x\cos x\,dx \le 2\pi^2$. **SCORE: _____ / 15 PTS**

FOR $0 \le x \le \frac{\pi}{3}$, $\frac{1}{2} \le \cos x \le 1$, ③

$18x \le 36x\cos x \le 36x$, ③

$$\int_0^{\frac{\pi}{3}} 18x\,dx \le \int_0^{\frac{\pi}{3}} 36x\cos x\,dx \le \int_0^{\frac{\pi}{3}} 36x\,dx \quad ③$$

$$② \; 9x^2 \Big|_0^{\frac{\pi}{3}} \le \int_0^{\frac{\pi}{3}} 36x\cos x\,dx \le 18x^2 \Big|_0^{\frac{\pi}{3}} \; ②$$

$$\pi^2 = 9\left(\frac{\pi^2}{9}\right) \le \int_0^{\frac{\pi}{3}} 36x\cos x\,dx \le 18\left(\frac{\pi^2}{9}\right) = 2\pi^2$$

① ①

<u>Using proper English and mathematical notation</u>, state all parts of the Fundamental Theorem of Calculus. (The Net Change Theorem is one of the parts.) **SCORE: _____ / 15 PTS**

SEE SOLUTION OF QUIZ 3

Find $\frac{d}{dx}(x^2\cosh^{-1}4x + \operatorname{sech}\sqrt[3]{x})$. **SCORE: _____ / 15 PTS**

You may use any hyperbolic identities or the derivatives of any hyperbolic functions without proving them.

$$2x\cosh^{-1}4x + x^2\frac{1}{\sqrt{(4x)^2-1}}(4) - \operatorname{sech}\sqrt[3]{x}\tanh\sqrt[3]{x}\cdot\frac{1}{3}x^{-\frac{2}{3}}$$

$$= 2x\cosh^{-1}4x + \frac{4x^2}{\sqrt{16x^2-1}} - \frac{\operatorname{sech}\sqrt[3]{x}\tanh\sqrt[3]{x}}{3\sqrt[3]{x^2}}$$

③ EACH

If f is continuous and $\int_{-1}^{5} f(t)\,dt = 6$, find $\int_{-1}^{2} (5+4f(3-2t))\,dt$. **SCORE: _____ / 15 PTS**

3 $u = 3-2t$, $t=2 \to u=-1$, $t=-1 \to u=5$

$\frac{du}{dt} = -2$

$dt = -\frac{1}{2}du$

2 $\int_5^{-1} -\frac{1}{2}(5+4f(u))\,du = \int_5^{-1} -\frac{5}{2}\,du - 2\int_5^{-1} f(u)\,du$ 2

2

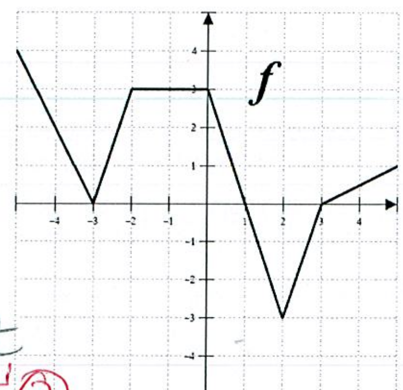
$= -\frac{5}{2}(-1-5) + 2\int_{-1}^{5} f(u)\,du$ 2

2

$= 15 + 2(6)$

$= 27$ 2

Let $g(x) = \int_{-3}^{x} f(t)\,dt$, where f is the function whose graph is shown on the right. **SCORE: _____ / 30 PTS**



[a] Find $g'(5)$. **Explain your answer very briefly.**

$g'(5) = f(5) = 1$

3

[b] Find $g(5)$.

$\int_{-3}^{5} f(t)\,dt = \int_{-3}^{1} f(t)\,dt + \int_{1}^{3} f(t)\,dt + \int_{3}^{5} f(t)\,dt$ 3

$= \frac{1}{2}(4+2)(3) - \frac{1}{2}(2)(3) + \frac{1}{2}(2)(1)$

$= 9 - 3 + 1 = 7$

2 2 1 1

[c] Find all inflection points of g. **Explain your answer very briefly.**

$g'(x) = f(x)$ CHANGES FROM DECREASING TO INCREASING

4

@ $x = -3, 2$

6

[d] Find all local minima of g. **Explain your answer very briefly.**

$g'(x) = f(x)$ CHANGES FROM NEGATIVE TO POSITIVE

4

@ $x = 3$

4

Evaluate the following integrals, or explain why they can't be evaluated. **SCORE: _____ / 60 PTS**

[a] $\displaystyle\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \frac{3-6\cos t}{\sin^2 t}\,dt$

15

$= \int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} (3\csc^2 t - 6\csc t\cot t)\,dt$ 5

$= (-3\cot t + 6\csc t)\Big|_{\frac{\pi}{6}}^{\frac{2\pi}{3}}$ 5

$= \left(-3\cdot\frac{-\sqrt{3}}{3} + 6\cdot\frac{2\sqrt{3}}{3}\right) - (-3\cdot\sqrt{3} + 6\cdot 2)$

$= (\sqrt{3} + 4\sqrt{3}) - (-3\sqrt{3} + 12)$

$= 8\sqrt{3} - 12$ 5

[b] $\displaystyle\int_{-\pi}^{\pi} \theta^2 \sec\theta\,d\theta$

10

$\theta^2\sec\theta$ IS DISCONTINUOUS 4
@ $\theta = \pm\frac{\pi}{2}$ 3
FTC DOES NOT APPLY 3

SEE ALTERNATE SOLUTION ON OTHER KEYS

[c] $\displaystyle\int_{-2}^{2} (10x-22)\sqrt{5-2x}\,dx$

20

3 $u = \sqrt{5-2x}$; $x = 2 \longrightarrow u = 1$; $x = -2 \longrightarrow u = 3$; $x = \frac{5-u^2}{2}$ 3

$\frac{du}{dx} = \frac{1}{2\sqrt{5-2x}}\cdot(-2)$

$= -\frac{1}{\sqrt{5-2x}}$

$dx = -\sqrt{5-2x}\,du$ 3

$(10x-22)\sqrt{5-2x}\,dx$

$= -(10x-22)(5-2x)\,du$

$= -\left(10\cdot\frac{5-u^2}{2} - 22\right)(u^2)\,du$

$= (5u^2-3)u^2\,du$

$= (5u^4-3u^2)\,du$ 3

3 $\int_3^1 (5u^4-3u^2)\,du = (u^5-u^3)\Big|_3^1 = (1^5-1^3)-(3^5-3^3) = -(243-27) = -216$ 3 2

[d] $\displaystyle\int_{-1}^{1} (y^2\tan y - 3\sqrt{1-y^2})\,dy = -\frac{3\pi}{2}$ 2

15

$= \int_{-1}^{1} y^2\tan y\,dy - 3\int_{-1}^{1}\sqrt{1-y^2}\,dy$ 3

$y^2\tan y$ IS CONTINUOUS ON $[-1,1]$ 2

$(-y)^2\tan(-y) = -y^2\tan y$ 2 ODD 2

SO $\int = 0$ 2

$\int_{-1}^{1}\sqrt{1-y^2}\,dy = \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$ 2